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Study of decay dynamics and CP asymmetry in $D^+ \rightarrow K_L^0 e^+ \nu_e$ decay

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Using 2.92 fb^{-1} of electron-positron annihilation data collected at $\sqrt{s} = 3.773 \text{ GeV}$ with the BESIII detector, we obtain the first measurements of the absolute branching fraction $\mathcal{B}(D^+ \rightarrow K_L^0 e^+ \nu_e) = (4.481 \pm 0.027(\text{stat.}) \pm 0.103(\text{sys.}))\%$ and the CP asymmetry $A_{CP}^{D^+ \rightarrow K_L^0 e^+ \nu_e} = (-0.59 \pm 0.60(\text{stat.}) \pm 1.48(\text{sys.}))\%$. From the $D^+ \rightarrow K_L^0 e^+ \nu_e$ differential decay rate distribution, the product of the hadronic form factor and the magnitude of the CKM matrix element, $f_+^K(0)|V_{cs}|$, is determined

to be $0.728 \pm 0.006(\text{stat.}) \pm 0.011(\text{sys.})$. Using $|V_{cs}|$ from the SM constrained fit with the measured $f_+^K(0)|V_{cs}|$, $f_+^K(0) = 0.748 \pm 0.007(\text{stat.}) \pm 0.012(\text{sys.})$ is obtained, and utilizing the unquenched LQCD calculation for $f_+^K(0)$, $|V_{cs}| = 0.975 \pm 0.008(\text{stat.}) \pm 0.015(\text{sys.}) \pm 0.025(\text{LQCD})$.

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I. INTRODUCTION

In the Standard Model (SM), violation of the combined charge-conjugation and parity symmetries (CP) arises from a nonvanishing irreducible phase in the Cabibbo-Kobayashi-Maskawa (CKM) flavor-mixing matrix [1, 2]. Although in the SM, CP violation in the charm sector is expected to be very small, $\mathcal{O}(10^{-3})$ or below [3], reference [4] finds that K^0 - \bar{K}^0 mixing will give rise to a clean CP violation signal of magnitude of $-2\text{Re}(\epsilon) \approx -3.3 \times 10^{-3}$ in the semileptonic decays $D^+ \rightarrow K_L^0(K_S^0)e^+\nu_e$.

Semileptonic decays of mesons allow determination of various important SM parameters, including elements of the CKM matrix, which in turn allows the physics of the SM to be tested at its most fundamental level. In the limit of zero electron mass, the differential decay rate for a D semileptonic decay with a pseudoscalar meson P is given by

$$\frac{d\Gamma(D \rightarrow Pe\nu_e)}{dq^2} = \frac{G_F^2 |V_{cs(d)}|^2}{24\pi^3} p^3 |f_+(q^2)|^2, \quad (1)$$

where G_F is the Fermi constant, $V_{cs(d)}$ is the relevant CKM matrix element, p is the momentum of the daughter meson in the rest frame of the parent D , $f_+(q^2)$ is the form factor, and q^2 is the invariant mass squared of the lepton-neutrino system.

In this paper, the first measurements of the absolute branching fraction and the CP asymmetry for the decay $D^+ \rightarrow K_L^0 e^+ \nu_e$, as well as the form-factor parameters for three different theoretical models that describe the weak hadronic charged currents in $D^+ \rightarrow K_L^0 e^+ \nu_e$ are presented. The paper is organized as follows: The BESIII detector and data sample are described in Sec. II. The analysis technique is introduced in Sec. III. In Secs. IV and V the measurements of the absolute branching fraction, the CP asymmetry and the form-factor parameters for the decay $D^+ \rightarrow K_L^0 e^+ \nu_e$ are described. Finally, a summary is provided in Sec. VI.

II. THE BESIII DETECTOR AND DATA SAMPLE

The analysis presented in this paper is based on a data sample with an integrated luminosity of 2.92 fb^{-1} [5] collected with the BESIII detector [6] at the center-of-mass energy of $\sqrt{s} = 3.773 \text{ GeV}$. The BESIII detector is a general-purpose detector at the BEPCII [7] double storage rings. The detector has a geometrical acceptance of 93% of the full solid angle. We briefly describe the components of BESIII from the interaction point (IP)

outwards. A small-cell multilayer drift chamber (MDC), using a helium-based gas to measure momenta and specific ionization of charged particles, is surrounded by a time-of-flight (TOF) system based on plastic scintillators which determines the time of flight of charged particles. A CsI(Tl) electromagnetic calorimeter (EMC) detects electromagnetic showers. These components are all situated inside a superconducting solenoid magnet, which provides a 1.0 T magnetic field parallel to the beam direction. Finally, a multilayer resistive plate counter system installed in the iron flux return yoke of the magnet is used to track muons. The momentum resolution for charged tracks in the MDC is 0.5% for a transverse momentum of $1 \text{ GeV}/c$. The energy resolution for showers in the EMC is 2.5% for 1 GeV photons. More details on the features and capabilities of BESIII can be found elsewhere [6].

The performance of the BESIII detector is simulated using a GEANT4-based [8] Monte Carlo (MC) program. To develop selection criteria and test the analysis technique, several MC samples are used. For the production of $\psi(3770)$, the KKMC [9] package is used; the beam energy spread and the effects of initial-state radiation (ISR) are included. Final-state radiation (FSR) of charged tracks is taken into account with the PHOTOS package [12]. $\psi(3770) \rightarrow D\bar{D}$ events are generated using EVTGEN [10, 11], and each D meson is allowed to decay according to the branching fractions in the Particle Data Group (PDG) [13]. We refer to this as the “generic MC.” The equivalent luminosity of the MC samples is about 10 times that of the data. A sample of $\psi(3770) \rightarrow D\bar{D}$ events, in which the D meson decays to the signal semileptonic mode and the \bar{D} decays to one of the hadronic final states used in the tag reconstruction, is referred to as the “signal MC”. In both the generic and signal MC samples, the semileptonic decays are generated using the modified pole parametrization [18] (see Sec. VB).

III. EVENT SELECTION

At the $\psi(3770)$ peak, $D\bar{D}$ pairs are produced. First, we select the single-tag (ST) sample in which a D^- is reconstructed in a hadronic decay mode. From the ST sample, the double-tag (DT) events of $D^+ \rightarrow K_L^0 e^+ \nu_e$ are selected. The numbers of the ST and DT events are given by

$$\begin{aligned} N_{\text{ST}} &= N_{D^+D^-} \mathcal{B}_{\text{tag}} \epsilon^{\text{ST}}, \\ N_{\text{DT}} &= N_{D^+D^-} \mathcal{B}_{\text{tag}} \mathcal{B}_{\text{sig}} \epsilon^{\text{DT}}, \end{aligned} \quad (2)$$

where $N_{D^+D^-}$ is the number of D^+D^- pairs produced, N_{ST} and N_{DT} are the numbers of the ST and DT events, ϵ_{ST} and ϵ_{DT} are the corresponding efficiencies, and \mathcal{B}_{tag} and \mathcal{B}_{sig} are the branching fractions of the hadronic tag decay and the signal decay. In this analysis, the charge-dependent branching fractions are measured, so there is no factor of two in Eq. (2). From Eq. (2), we obtain

$$\mathcal{B}_{sig} = \frac{N_{DT}/\epsilon_{DT}}{N_{ST}/\epsilon_{ST}} = \frac{N_{DT}/\epsilon}{N_{ST}}, \quad (3)$$

where $\epsilon = \epsilon_{DT}/\epsilon_{ST}$ is the efficiency of finding a signal candidate in the presence of a ST D , which is obtained from generic MC simulations.

A. Selection of ST events

Each charged track is required to satisfy $|\cos\theta| < 0.93$, where θ is the polar angle with respect to the beam axis. Charged tracks other than those from the K_S^0 are required to have their points of closest approach to the beamline within 10 cm from the IP along the beam axis and within 1 cm in the plane perpendicular to the beam axis. Particle identification for charged hadrons h ($h = \pi, K$) is accomplished by combining the measured energy loss (dE/dx) in the MDC and the flight time obtained from the TOF to form a likelihood $\mathcal{L}(h)$ for each hadron hypothesis. The K^\pm (π^\pm) candidates are required to satisfy $\mathcal{L}(K) > \mathcal{L}(\pi)$ ($\mathcal{L}(\pi) > \mathcal{L}(K)$).

The K_S^0 candidates are selected from pairs of oppositely charged tracks which satisfy a vertex-constrained fit to a common vertex. The vertices are required to be within 20 cm of the IP along the beam direction; no constraint in the transverse plane is applied. Particle identification is not required, and the two charged tracks are assumed to be pions. We require $|M_{\pi^+\pi^-} - M_{K_S^0}| < 12 \text{ MeV}/c^2$, where $M_{K_S^0}$ is the nominal K_S^0 mass [13] and $12 \text{ MeV}/c^2$ is about 3 standard deviations of the observed K_S^0 mass resolution. Lastly, the K_S^0 candidate must have a decay length more than 2 standard deviations of the vertex resolution away from the IP.

Reconstructed EMC showers that are separated from the extrapolated positions of any charged tracks by more than 10° are taken as photon candidates. The energy deposited in the nearby TOF counters is included to improve the reconstruction efficiency and energy resolution. Photon candidates must have a minimum energy of 25 MeV for barrel showers ($|\cos\theta| < 0.80$) and 50 MeV for end-cap showers ($0.86 < |\cos\theta| < 0.92$). The shower timing is required to be no later than 700 ns after the reconstructed event start time to suppress electronic noise and energy deposits unrelated to the event.

The π^0 candidates are reconstructed from pairs of photons, and the invariant mass $M_{\gamma\gamma}$ is required to satisfy $0.110 < M_{\gamma\gamma} < 0.155 \text{ GeV}/c^2$. The invariant mass of two photons is constrained to the nominal π^0 mass [13] by a kinematic fit, and the χ^2 of the kinematic fit is required to be less than 20.

We form D^\pm candidates decaying into final hadronic states of $K^\mp\pi^\pm\pi^\pm$, $K^\mp\pi^\pm\pi^\pm\pi^0$, $K_S^0\pi^\pm\pi^0$, $K_S^0\pi^\pm\pi^\pm\pi^\mp$, $K_S^0\pi^\pm$, and $K^+K^-\pi^\pm$. Two variables are used to identify valid ST D candidates: $\Delta E \equiv E_D - E_{beam}$, the energy difference between the energy of the ST D (E_D) and the beam energy (E_{beam}), and the beam-constrained mass $M_{BC} \equiv \sqrt{E_{beam}^2/c^4 - |\vec{p}_D|^2/c^2}$, where \vec{p}_D is the momentum of the D . The ST D signal should peak at the nominal D mass in the M_{BC} distribution and around zero in the ΔE distribution. We only accept one candidate per mode; when multiple candidates are present in an event, the one with the smallest $|\Delta E|$ is kept. Backgrounds are suppressed by the mode-dependent ΔE requirements listed in Table I.

TABLE I. Requirements on ΔE for the ST D candidates. The limits are set at approximately 3 standard deviations of the ΔE resolution.

Mode	Requirement (GeV)
$D^\pm \rightarrow K^\mp\pi^\pm\pi^\pm$	$-0.030 < \Delta E < 0.030$
$D^\pm \rightarrow K^\mp\pi^\pm\pi^\pm\pi^0$	$-0.052 < \Delta E < 0.039$
$D^\pm \rightarrow K_S^0\pi^\pm\pi^0$	$-0.057 < \Delta E < 0.040$
$D^\pm \rightarrow K_S^0\pi^\pm\pi^\pm\pi^\mp$	$-0.034 < \Delta E < 0.034$
$D^\pm \rightarrow K_S^0\pi^\pm$	$-0.032 < \Delta E < 0.032$
$D^\pm \rightarrow K^+K^-\pi^\pm$	$-0.030 < \Delta E < 0.030$

The ST yields of data are determined by binned maximum likelihood fits to the M_{BC} distributions. The signal MC line shape is used to describe the D signal, and an ARGUS [14] function is used to model the combinatorial backgrounds from the continuum light hadron production, $\gamma_{ISR}\psi(3686)$, $\gamma_{ISR}J/\psi$ and non-signal $D\bar{D}$ decays. A Gaussian function, with the standard deviation and the central value as free parameters, is convoluted with the line shape to account for imperfect modeling of the detector resolution and beam energy.

The charge-conjugated tag modes are fitted simultaneously, with the same signal and ARGUS background shapes for the tag and charge conjugated modes. The numbers of signal and background events are left free. Figures 1 and 2 show the fits to the M_{BC} distributions of the ST D^+ and D^- candidates in data, respectively. The ST yields are obtained by integrating the fitted signal function in the narrower M_{BC} signal region ($1.86 < M_{BC} < 1.88 \text{ GeV}/c^2$) and are listed in Table II.

B. Selection of DT events

After ST D candidates are identified, we search for electrons and K_L^0 showers among the unused charged tracks and neutral showers. For electron identification, the ratio $\mathcal{R}'_{\mathcal{L}'(e)} \equiv \mathcal{L}'(e)/[\mathcal{L}'(e) + \mathcal{L}'(\pi) + \mathcal{L}'(K)]$ is required to be greater than 0.8, where the likelihood $\mathcal{L}'(i)$ for the hypothesis $i = e, \pi$ or K is formed by combining the EMC information with the dE/dx and

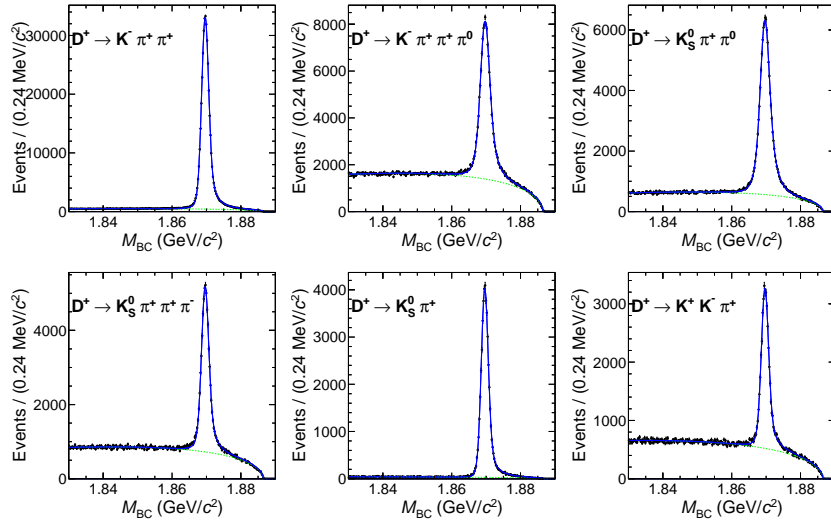


FIG. 1. Fits to the M_{BC} distributions of the ST D^+ candidates for data. The dots with error bars are for data, and the blue solid curves are the results of the fits. The green dashed curves are the fitted backgrounds.

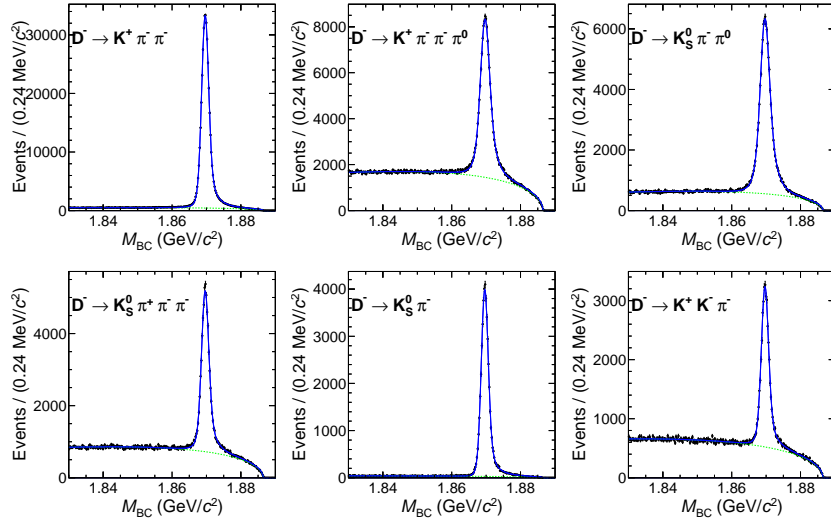


FIG. 2. Fits to the M_{BC} distributions of the ST D^- candidates for data. The dots with error bars are data, and the blue solid curves are the results of the fits. The green dashed curves are the fitted backgrounds.

TOF information. The energy lost by electrons to bremsstrahlung photons is partially recovered by adding the energy of showers that are within 5° of the electron and are not matched to other charged particles. The selected electron is required to have the opposite charge from the ST D . Events that include charged tracks other than those of the ST D and the electron are vetoed.

Because of the long K_L^0 lifetime, very few K_L^0 decay in the MDC. However, most K_L^0 will interact in the material of the EMC, which gives their position, and deposit part of their energy. We search for K_L^0 candidates by reconstructing all other particles in the event; we then loop over unused reconstructed neutral showers, taking the direction to the shower as the flight direction

of the K_L^0 . Using energy-momentum conservation and the constraint $U_{\text{miss}} = 0$, we calculate the momentum magnitude $|\vec{p}_{K_L^0}|$ of the K_L^0 and the four-vector of the unreconstructed neutrino in the event. The variable U_{miss} is expected to peak at zero for semileptonic decay candidates and is defined as

$$U_{\text{miss}} \equiv E_{\text{miss}} - c|\vec{p}_{\text{miss}}|, \quad (4)$$

where

$$\begin{aligned} E_{\text{miss}} &= E_{\text{tot}} - E_{\text{tag}} - E_{K_L^0} - E_e, \\ \vec{p}_{\text{miss}} &= \vec{p}_{\text{tot}} - \vec{p}_{\text{tag}} - \vec{p}_{K_L^0} - \vec{p}_e; \end{aligned} \quad (5)$$

E_{tot} , E_{tag} , $E_{K_L^0}$ and E_e are the energies of the e^+e^- , the ST D , the K_L^0 and the electron; \vec{p}_{tot} , \vec{p}_{tag} , $\vec{p}_{K_L^0}$ and

\vec{p}_e refer to their momenta. $E_{K_L^0}$ is calculated by $E_{K_L^0} = \sqrt{|\vec{p}_{K_L^0}|^2 + m_{K_L^0}^2}$. In order to suppress background from fake photons, the energy of K_L^0 shower should be greater than 0.1 GeV. We also reject photons that may come from π^0 's by rejecting γ in any $\gamma\gamma$ combination with $0.110 < M_{\gamma\gamma} < 0.155 \text{ GeV}/c^2$. In events with multiple K_L^0 shower candidates, the most energetic shower is chosen. The inferred four-momentum of the K_L^0 is used to determine the reconstructed q^2 , the invariant mass squared of the $e^+\nu_e$ pair, by

$$q^2 = \frac{1}{c^4}(E_{\text{tot}} - E_{\text{tag}} - E_{K_L^0})^2 - \frac{1}{c^2}|\vec{p}_{\text{tot}} - \vec{p}_{\text{tag}} - \vec{p}_{K_L^0}|^2. \quad (6)$$

Similar to the determination of the ST yields, we obtain the DT yields of data from the fits to the M_{BC} distributions of the corresponding ST D candidates. Figures 3 and 4 show the fits to the M_{BC} distributions of the DT D^+ and D^- candidates in data, respectively. From the fits, we obtain the DT yields in data, which are listed in the third column of Table II.

C. Estimation of backgrounds

The K_L^0 reconstruction efficiencies of data and MC differ, so the K_L^0 reconstruction efficiency of the generic MC is corrected to that of data. The correction factors of K_L^0 reconstruction efficiencies are determined from two control samples ($J/\psi \rightarrow K^*(892)^\pm K^\mp$ with $K^*(892)^\pm \rightarrow K_L^0\pi^\pm$ and $J/\psi \rightarrow \phi K_L^0 K^\pm \pi^\mp$), which are described in Appendix A. The corrected generic MC samples are used to determine the amount of peaking background and the efficiency for $D^+ \rightarrow K_L^0 e^+ \nu_e$.

We examine the topologies of the corrected generic MC samples to study the composition of the DT samples. In the M_{BC} signal region, the DT D candidates can be divided into the following categories:

- Signal: Tag-side and signal-side correctly matched.
- Background:
 - Tag-side mismatched events (Bkg I).
 - Tag-side matched but signal-side mismatched signal events (Bkg II).
 - Tag-side matched but $D \rightarrow X e \nu_e$ non-signal events on signal side (Bkg III).
 - Tag-side matched but $D \rightarrow X \mu \nu_\mu$ events on signal side (Bkg IV).
 - Tag-side matched but non-leptonic D decay events on signal side (Bkg V).

In the selected DT candidates, the proportion of signal events varies from 49% to 58% according to the specific hadronic tag mode. Bkg I comes from $D\bar{D}$ decays in which the hadronic tag D is mis-reconstructed and non- $D\bar{D}$ processes, and varies from 1% to 12% according

to the specific hadronic tag mode. Bkg II ($\sim 10\%$) consists of $D^+ \rightarrow K_L^0 e^+ \nu_e$ events of which K_L^0 shower is mis-reconstructed. The dominant background in the DT sample is Bkg III ($\sim 24\%$), which is from $D^+ \rightarrow \bar{K}^*(892)^0 e^+ \nu_e$ (41.9%), $D^+ \rightarrow K_S^0 e^+ \nu_e$ (41.2%), $D^+ \rightarrow \pi^0 e^+ \nu_e$ (10.2%), $D^+ \rightarrow \eta e^+ \nu_e$ (6.0%) and $D^+ \rightarrow \omega e^+ \nu_e$ (0.7%). Bkg IV ($\sim 3\%$) consists of $D^+ \rightarrow K_L^0 \mu^+ \nu_\mu$ (65.2%), $D^+ \rightarrow \bar{K}^*(892)^0 \mu^+ \nu_\mu$ (23.3%) and $D^+ \rightarrow K_S^0 \mu^+ \nu_\mu$ (11.5%). Bkg V ($\sim 3\%$) consists of $D^+ \rightarrow \bar{K}^0 \pi^+ \pi^0$ (78%) and $D^+ \rightarrow \bar{K}^0 K^*(892)^+$ (22%).

IV. BRANCHING FRACTION AND CP ASYMMETRY

The branching fraction for $D^+ \rightarrow K_L^0 e^+ \nu_e$ (\mathcal{B}_{sig}) is determined by

$$\mathcal{B}_{\text{sig}} = \frac{N_{\text{DT}}(1 - f_{\text{bkg}}^{\text{peak}})}{\epsilon N_{\text{ST}}}, \quad (7)$$

where N_{DT} , N_{ST} are the DT and ST yields, $f_{\text{bkg}}^{\text{peak}}$ is the proportion of peaking backgrounds in the DT candidates (from Bkg II to Bkg V), ϵ is the efficiency for finding $D^+ \rightarrow K_L^0 e^+ \nu_e$ in the presence of ST D . $f_{\text{bkg}}^{\text{peak}}$ and ϵ are obtained from the K_L^0 efficiency corrected generic MC samples. The $D^+ \rightarrow K_L^0 e^+ \nu_e$ branching fractions for different ST modes are listed in Table II. We obtain $\mathcal{B}(D^+ \rightarrow K_L^0 e^+ \nu_e) = (4.454 \pm 0.038 \pm 0.102)\%$ and $\mathcal{B}(D^- \rightarrow K_L^0 e^- \bar{\nu}_e) = (4.507 \pm 0.038 \pm 0.104)\%$, which are the weighted averages of the six ST modes for D^+ and D^- separately. Combining these branching fractions, we obtain the averaged branching fraction $\bar{\mathcal{B}}(D^+ \rightarrow K_L^0 e^+ \nu_e) = (4.481 \pm 0.027 \pm 0.103)\%$, which agrees well with the measurement of $\mathcal{B}(D^+ \rightarrow K_S^0 e^+ \nu_e)$ of CLEO-c [15]. The CP asymmetry of $D^+ \rightarrow K_L^0 e^+ \nu_e$ is

$$A_{CP} \equiv \frac{\mathcal{B}(D^+ \rightarrow K_L^0 e^+ \nu_e) - \mathcal{B}(D^- \rightarrow K_L^0 e^- \bar{\nu}_e)}{\mathcal{B}(D^+ \rightarrow K_L^0 e^+ \nu_e) + \mathcal{B}(D^- \rightarrow K_L^0 e^- \bar{\nu}_e)} \quad (8)$$

$$= (-0.59 \pm 0.60 \pm 1.48)\%.$$

This result is consistent with the theoretical prediction in Ref. [4] (-3.3×10^{-3}).

Table III summarizes the systematic uncertainties in the measurements of absolute branching fractions and the CP asymmetry of $D^+ \rightarrow K_L^0 e^+ \nu_e$. A brief description of each systematic uncertainty is provided below.

1. Electron (positron) track-finding and identification (ID) efficiency

Uncertainties of electron (positron) track-finding and ID efficiency are obtained by comparing the track-finding and ID efficiencies for the electrons (positrons) from radiative Bhabha processes in the data and MC. Considering both the $\cos\theta$, where θ is the polar angle of the positron, and momentum distributions of the electrons (positrons) of the signal events, we obtain the two-dimensional weighted

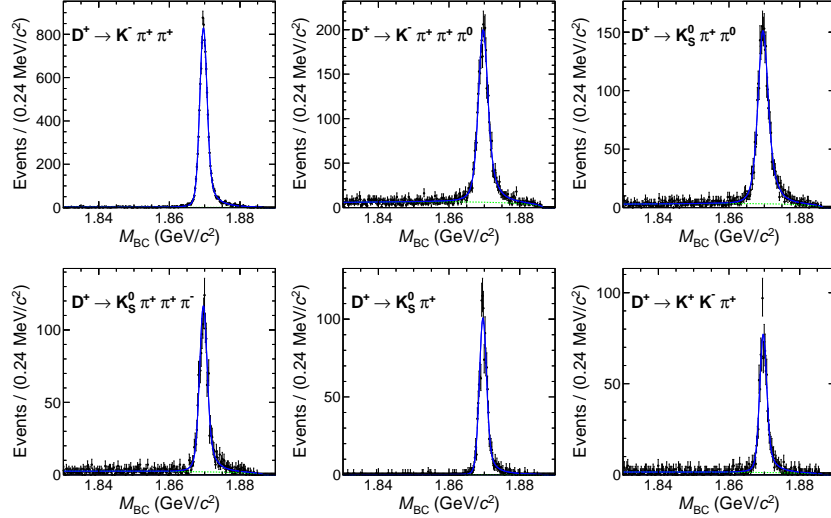


FIG. 3. Fits to the M_{BC} distributions of the DT D^+ candidates for data. The dots with error bars are for data, and the blue solid curves are the results of the fits. The green dashed curves are the fitted combinatorial backgrounds.

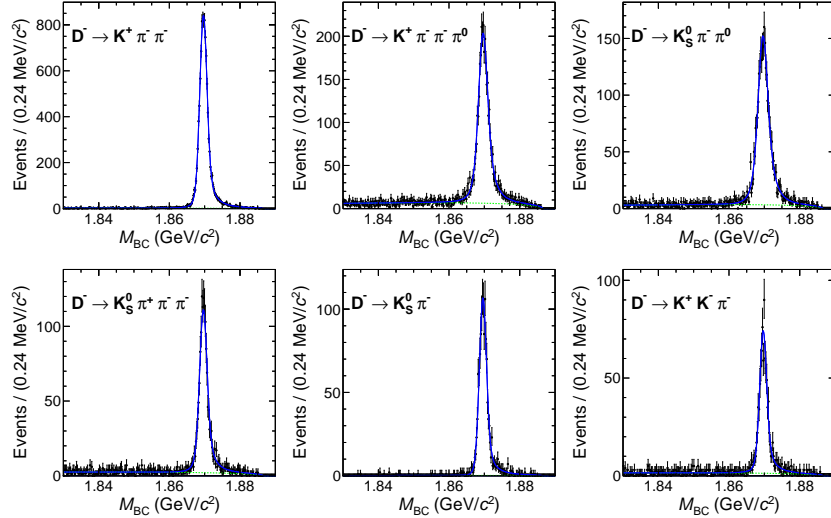


FIG. 4. Fits to the M_{BC} distributions of the DT D^- candidates for data. The dots with error bars are for data, and the blue solid curves are the results of the fits. The green dashed curves are the fitted combinatorial backgrounds.

uncertainty of electron (positron) track-finding to be 0.5%, and the averaged uncertainties of positron and electron ID efficiency to be 0.03% and 0.10%, respectively.

2. K_L^0 efficiency correction

We take the relative statistical uncertainty of the K_L^0 efficiency difference between data and MC as a function of momentum (as shown in Fig. 7 in Appendix A) as the uncertainty of the K_L^0 efficiency correction. Weighting these uncertainties by the K_L^0 momentum distribution in $D^+ \rightarrow K_L^0 e^+ \nu_e$, we obtain the uncertainties of the $K^0 \rightarrow K_L^0$ and $\bar{K}^0 \rightarrow K_L^0$ efficiency corrections to both be 1.2%.

3. Extra χ^2 cut for K_L^0 efficiency correction

As described in Appendix A, in the determination of correction factor of the K_L^0 efficiency, we apply a χ^2 cut which brings an extra uncertainty. The uncertainty of the χ^2 cut is obtained by comparing the cut efficiency between data and MC using two control samples ($J/\psi \rightarrow K^*(892)^\pm K^\mp$ with $K^*(892)^\pm \rightarrow K_L^0 \pi^\pm$ and $J/\psi \rightarrow \phi K_L^0 K^\pm \pi^\mp$). Weighting by the momentum distribution of the K_L^0 of signal events, the uncertainty of the extra χ^2 cut ($\chi^2 < 100$) is 0.8%.

4. Peaking backgrounds in DT

For Bkg II, from Eq. (7) the ratio of mis-

TABLE II. Summary of the ST yields (N_{ST}), the DT yields (N_{DT}), the peaking background rates for the DT candidates ($f_{\text{bkg}}^{\text{peak}}$), the detection efficiency (ϵ) and the branching fraction for signal decay for each ST mode (\mathcal{B}_{sig}). The averages are the weighted average of the individual ST mode branching fractions. The uncertainties are statistical.

$D^+ \rightarrow K_L^0 e^+ \nu_e$					
Tag Mode	N_{ST}	N_{DT}	$f_{\text{bkg}}^{\text{peak}}(\%)$	$\epsilon(\%)$	$\mathcal{B}_{\text{sig}}(\%)$
$D^- \rightarrow K^+ \pi^- \pi^-$	410200 ± 670	10492 ± 103	41.83 ± 0.28	33.96 ± 0.10	4.381 ± 0.050
$D^- \rightarrow K^+ \pi^- \pi^- \pi^0$	120060 ± 457	3324 ± 64	44.78 ± 0.49	33.14 ± 0.19	4.613 ± 0.103
$D^- \rightarrow K_S^0 \pi^- \pi^0$	102136 ± 378	2658 ± 56	38.93 ± 0.58	35.67 ± 0.21	4.456 ± 0.108
$D^- \rightarrow K_S^0 \pi^- \pi^- \pi^+$	59158 ± 303	1459 ± 41	40.84 ± 0.76	32.51 ± 0.27	4.488 ± 0.145
$D^- \rightarrow K_S^0 \pi^-$	47921 ± 225	1287 ± 36	38.90 ± 0.88	35.07 ± 0.32	4.679 ± 0.155
$D^- \rightarrow K^+ K^- \pi^-$	35349 ± 239	905 ± 32	44.64 ± 0.97	30.98 ± 0.35	4.575 ± 0.190
Average					4.454 ± 0.038
$D^- \rightarrow K_L^0 e^- \bar{\nu}_e$					
Tag Mode	N_{ST}	N_{DT}	$f_{\text{bkg}}^{\text{peak}}(\%)$	$\epsilon(\%)$	$\mathcal{B}_{\text{sig}}(\%)$
$D^+ \rightarrow K^- \pi^+ \pi^+$	407666 ± 668	10354 ± 103	40.44 ± 0.29	34.02 ± 0.11	4.447 ± 0.051
$D^+ \rightarrow K^- \pi^+ \pi^+ \pi^0$	117555 ± 450	3264 ± 63	42.28 ± 0.52	33.19 ± 0.19	4.829 ± 0.107
$D^+ \rightarrow K_S^0 \pi^+ \pi^0$	101824 ± 378	2642 ± 55	39.06 ± 0.58	35.92 ± 0.21	4.402 ± 0.104
$D^+ \rightarrow K_S^0 \pi^+ \pi^+ \pi^-$	59046 ± 303	1533 ± 42	39.68 ± 0.77	33.44 ± 0.27	4.683 ± 0.147
$D^+ \rightarrow K_S^0 \pi^+$	48240 ± 226	1217 ± 35	38.50 ± 0.88	35.20 ± 0.32	4.408 ± 0.147
$D^+ \rightarrow K^+ K^- \pi^+$	35742 ± 240	942 ± 32	44.04 ± 0.95	32.40 ± 0.36	4.552 ± 0.181
Average					4.507 ± 0.038

TABLE III. Systematic uncertainties in the measurements of absolute branching fraction and the CP asymmetry of $D^+ \rightarrow K_L^0 e^+ \nu_e$.

Source	$D^+ \rightarrow K_L^0 e^+ \nu_e(\%)$	$D^- \rightarrow K_L^0 e^- \bar{\nu}_e(\%)$
Electron tracking	0.5	0.5
Electron ID	0.1	0.1
K_L^0 efficiency correction	1.2	1.2
Extra χ^2 cut for K_L^0 efficiency correction	0.8	0.8
Peaking backgrounds in DT	1.6	1.6
M_{BC} fit	negligible	negligible
Total (Branching fraction)	2.3	2.3
Total (CP asymmetry)	2.1	2.1

reconstructed K_L^0 will not affect the measured branching fraction, since the numerator and the denominator share the common factor. The uncertainties of the peaking backgrounds of mis-reconstructed K_L^0 can be safely ignored. For Bkg III, Bkg IV and Bkg V, we determine the change of the number of DT events by varying the branching fractions of peaking background channels by 1σ , and the uncertainty of peaking backgrounds in DT events is 1.6%.

5. M_{BC} fit

To evaluate the systematic uncertainty from the M_{BC} fit, we determine the changes of the DT yields divided by the ST yields when varying the standard deviation of the convoluted Gaussian function by $\pm 1\sigma$ deviation for each tag mode. We find that

they are negligible.

The total systematic uncertainties of the branching fractions for $D^+ \rightarrow K_L^0 e^+ \nu_e$ and $D^- \rightarrow K_L^0 e^- \bar{\nu}_e$ are determined to be 2.3% and 2.3%, respectively, by adding all contributions in quadrature. In the determination of the CP asymmetry, the corresponding systematic uncertainties of branching fractions for $D^+ \rightarrow K_L^0 e^+ \nu_e$ and $D^- \rightarrow K_L^0 e^- \bar{\nu}_e$ are obtained in a similar fashion, except that the contribution of the extra χ^2 cut of K_L^0 efficiency correction is not used since it cancels. The systematic uncertainties entering the CP asymmetry are found to be 2.1% and 2.1%, respectively.

V. HADRONIC FORM FACTOR

A. Method of extraction of form factor

The number of produced signal events for each tag mode from the whole q^2 range can be written as

$$n = 2N_{D^+D^-} \mathcal{B}_{\text{tag}} \mathcal{B}_{\text{sig}} = N_{\text{tag}} \frac{\Gamma_{\text{sig}}}{\Gamma_{D^+}}, \quad (9)$$

where Γ_{sig} is the partial decay width of $D^+ \rightarrow K_L^0 e^+ \nu_e$ while Γ_{D^+} is the total decay width of D^+ . So we obtain

$$dn = \frac{N_{\text{tag}}}{\Gamma_{D^+}} d\Gamma_{\text{sig}} = N_{\text{tag}} \tau_{D^+} d\Gamma_{\text{sig}}, \quad (10)$$

where $\tau_{D^+} = 1/\Gamma_{D^+}$ is the D^+ lifetime and $d\Gamma_{\text{sig}}$ is the differential decay width of the signal.

Substituting Eq. (10) into Eq. (1), Eq. (1) can be rewritten as

$$\frac{dn}{dq^2} = AN_{\text{tag}} p^3 |f_+(q^2)|^2, \quad (11)$$

where $A = \frac{1}{2} \frac{G_F^2 |V_{cs}|^2}{24\pi^3} \tau_{D^+}$, and the number of observed semileptonic signal events as a function of q^2 is given by

$$\frac{dn_{\text{observed}}}{dq^2} = AN_{\text{tag}} [p^3(q'^2) |f_+(q'^2)|^2 \epsilon(q'^2)] \otimes \sigma(q'^2, q^2), \quad (12)$$

where q'^2 refers to the true value and q^2 refers to the measured value; $p(q'^2)$ is the momentum of K_L^0 in the rest frame of the parent D ; $\epsilon(q'^2)$ is the detection efficiency and $\sigma(q'^2, q^2)$ is the detector resolution. To account for detector effects, we use the theoretical function convoluted with a Gaussian detector resolution to describe the observed signal curve.

B. Form-factor parametrizations

The goal of any particular parametrization $f_+(q^2)$ of the semileptonic form factors is to provide an accurate, and physically meaningful, expression of the strong dynamics in the decays. One possible way to achieve this goal is to express the form factors in terms of a dispersion relation. This approach of using dispersion relations and dispersive bounds in the description of form factors, has been well established in the literature. In general, the dispersive representation is derived from the evaluation of the two point function [16, 17] and can be written as

$$f_+(q^2) = \frac{f_+(0)}{(1-\alpha)} \frac{1}{1-\frac{q^2}{m_{\text{pole}}^2}} + \frac{1}{\pi} \int_{(m_D+m_P)^2}^{\infty} \frac{\text{Im}f_+(t)}{t-q^2-i\epsilon} dt, \quad (13)$$

where m_D and m_P are the masses of the D meson and pseudoscalar meson respectively, while m_{pole} is the mass of the lowest-lying $c\bar{q}$ vector meson, with $c \rightarrow q$ the

quark transition of the semileptonic decay. For the charm semileptonic decays we have $m_{\text{pole}} = m_{D_s^*}$ for $D \rightarrow K e \nu_e$ decays. The parameter α expresses the size of the vector meson pole contribution to $f_+(0)$. It is common to write the contribution from the continuum integral as a sum of effective poles

$$f_+(q^2) = \frac{f_+(0)}{(1-\alpha)} \frac{1}{1-\frac{q^2}{m_{\text{pole}}^2}} + \sum_{k=1}^N \frac{\rho_k}{1-\frac{1}{\gamma_k} \frac{q^2}{m_{\text{pole}}^2}}, \quad (14)$$

where ρ_k and γ_k are expansion parameters.

The simplest parametrization, known as the simple pole model, assumes that the sum in Eq. (14) is dominated by a single pole

$$f_+(q^2) = \frac{f_+(0)}{1-\frac{q^2}{m_{\text{pole}}^2}}, \quad (15)$$

where the value of m_{pole} is predicted to be $m_{D_s^*}$. In experiments, m_{pole} is left as a free fit parameter to improve the fit quality.

Another parametrization is known as the modified pole model, or Becirevic-Kaidelov (BK) parametrization [18]. The idea is to add the first term in the effective pole expansion, while making simplifications such that the form factor can be determined with only two parameters: the intercept $f_+(0)$ and an additional shape parameter α . The simplified one-term expansion is usually written in the form

$$f_+(q^2) = \frac{f_+(0)}{(1-\frac{q^2}{m_{\text{pole}}^2})(1-\alpha\frac{q^2}{m_{\text{pole}}^2})}. \quad (16)$$

A third parametrization is known as the series expansion [19]. Exploiting the analytic properties of $f_+(q^2)$, a transformation of variables is made that maps the cut in the q^2 plane onto a unit circle $|z| < 1$, where

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad (17)$$

$t_{\pm} = (m_D \pm m_P)^2$, and t_0 is any real number less than t_+ . This transformation amounts to expanding the form factor about $q^2 = t_0$, with the expanded form factor given by

$$f_+(q^2) = \frac{1}{P(q^2)\phi(q^2, t_0)} \sum_{k=0}^{\infty} a_k(t_0) [z(q^2, t_0)]^k, \quad (18)$$

where a_k are real coefficients, $P(q^2) = z(q^2, M_{D_s^*}^2)$ for kaon final states, $P(q^2) = 1$ for pion final states, and $\phi(q^2, t_0)$ is any function that is analytic outside a cut in the complex q^2 plane that lies along the x -axis from t_+ to ∞ . This expansion has improved convergence properties over Eq. (14) due to the smallness of z ; for example, taking the traditional choice of $t_0 = t_+(1 -$

$(1 - t_-/t_+)^{1/2}$), which minimizes the maximum value of $z(q^2, t_0)$. Further, taking the standard choice of ϕ :

$$\begin{aligned} \phi(q^2, t_0) = & \sqrt{\frac{\pi m_c^2}{3}} \left(\frac{z(q^2, 0)}{-q^2} \right)^{5/2} \left(\frac{z(q^2, t_0)}{t_0 - q^2} \right)^{-1/2} \\ & \times \left(\frac{z(q^2, t_-)}{t_- - q^2} \right)^{-3/4} \frac{t_+ - q^2}{(t_+ - t_0)^{1/4}}, \end{aligned} \quad (19)$$

where m_c is the mass of charm quark, it can be shown that the sum over all k of a_k^2 is of order unity.

In practical use of the series expansion form factor, one often takes $k = 1$ and $k = 2$ in Eq. (18), which gives following two forms of the form factor.

- 2 par. series expansion of form factor is given by

$$f_+(q^2) = \frac{1}{P(q^2)\phi(q^2, t_0)} a_0(t_0) (1 + r_1(t_0)[z(q^2, t_0)]). \quad (20)$$

It can be rewritten as

$$\begin{aligned} f_+(q^2) = & \frac{1}{P(q^2)\phi(q^2, t_0)} \frac{f_+(0)P(0)\phi(0, t_0)}{1 + r_1(t_0)z(0, t_0)} \\ & \times (1 + r_1(t_0)[z(q^2, t_0)]), \end{aligned} \quad (21)$$

where $r_1 = a_1/a_0$.

- 3 par. series expansion of form factor is given by

$$\begin{aligned} f_+(q^2) = & \frac{1}{P(q^2)\phi(q^2, t_0)} a_0(t_0) \\ & \times (1 + r_1(t_0)[z(q^2, t_0)] + r_2(t_0)[z(q^2, t_0)]^2). \end{aligned} \quad (22)$$

It can be rewritten as

$$\begin{aligned} f_+(q^2) = & \frac{1}{P(q^2)\phi(q^2, t_0)} \frac{f_+(0)P(0)\phi(0, t_0)}{1 + r_1(t_0)z(0, t_0) + r_2(t_0)z^2(0, t_0)} \\ & \times (1 + r_1(t_0)[z(q^2, t_0)] + r_2(t_0)[z(q^2, t_0)]^2), \end{aligned} \quad (23)$$

where $r_1 = a_1/a_0$, $r_2 = a_2/a_0$.

C. Determination of $f_+^K(0)|V_{cs}|$

We perform simultaneous fits to the distributions of observed DT candidates as a function of q^2 for the six ST modes to determine $f_+^K(0)|V_{cs}|$. In the fits, we treat D^+ and D^- DT candidates together. The detection efficiency $\epsilon(q'^2)$ and detector resolution $\sigma(q'^2, q^2)$ are obtained from the K_L^0 efficiency corrected signal MC simulations. For each ST mode, $\epsilon(q'^2)$ is described by a fourth-order polynomial; the $(q^2 - q'^2)$ distribution is described by a Gaussian function. As an example, Figure 5 shows the fits to $\epsilon(q'^2)$ for signal events tagged by $D^\pm \rightarrow K^\mp \pi^\pm \pi^\pm$.

Simultaneous fits are made with one or two common parameters related to the form-factor shape to the data for the simple pole model (m_{pole}), the modified pole model (α), two-parameter series expansion (r_1) and

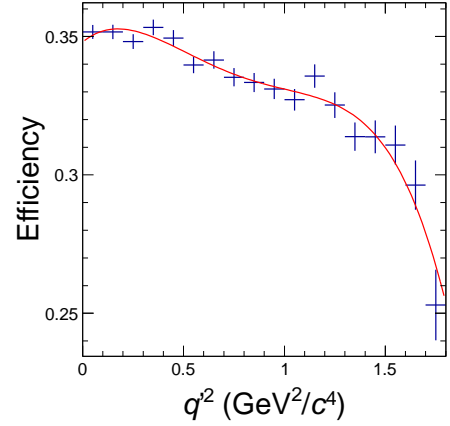


FIG. 5. Detection efficiency $\epsilon(q'^2)$ for signal events tagged by $D^\pm \rightarrow K^\mp \pi^\pm \pi^\pm$. The dots with error bars are the corrected signal MC efficiencies, and the curve is the fit result.

three-parameter series expansion (r_1, r_2). As an example, Figure 6 shows the simultaneous fit results using the two-parameter series expansion model. The signal PDF is constructed in the form of Eq. (12). For the background shape, as mentioned in Section III C, the shape and the number of Bkg I events are fixed according to the side-band region of the M_{BC} distribution ($1.83 < M_{\text{BC}} < 1.85 \text{ GeV}/c^2$) from data; for Bkgs from II to V, the shape is determined from the K_L^0 efficiency corrected generic MC samples. We also fix the relative proportion of N_{sig} , $N_{\text{Bkg II}}$ and $N_{\text{Bkg III}} + N_{\text{Bkg IV}}$ events, to the result from the K_L^0 efficiency corrected generic MC. Here, N_{sig} , $N_{\text{Bkg II}}$, $N_{\text{Bkg III}}$ and $N_{\text{Bkg IV}}$ represent the number of the signal, Bkg II, Bkg III and Bkg IV events, respectively.

The product $f_+^K(0)|V_{cs}|$ is obtained from

$$f_+^K(0)|V_{cs}| = \sqrt{\frac{48\pi^3}{G_F^2} \frac{N_{\text{sig}}}{N_{\text{tag}} \tau_{D^+} I}}, \quad (24)$$

where $I = \int [p^3(q'^2)|f_+(q'^2)|^2 \epsilon(q'^2)] \otimes \sigma(q'^2, q^2) dq'^2$.

Since the q^2 distribution of the signal events is smooth, the form-factor fit is insensitive to the detector resolution. For each tag mode, we use the full width at half maximum (FWHM) of the $(q^2 - q'^2)$ distribution to estimate $\sigma(q'^2, q^2)$ and obtain $\text{FWHM} = 0.0360 \text{ GeV}^2/c^4$ and the corresponding resolution $\sigma = \text{FWHM}/2\sqrt{2\ln 2} = 0.0153 \text{ GeV}^2/c^4$. The distributions of DT candidates as a function of q^2 are fit again by different models with the detector resolution $\sigma = 0.0153 \text{ GeV}^2/c^4$. Compared to the previous results, the form-factor parameters and the signal yields are almost unchanged. So the uncertainty of the detector resolution can be ignored in the form-factor fit.

Systematic uncertainties of the form-factor parameters are more sensitive to the distribution of backgrounds in this analysis. We use different side-band region of the M_{BC} distribution ($1.835 < M_{\text{BC}} < 1.855 \text{ GeV}/c^2$)

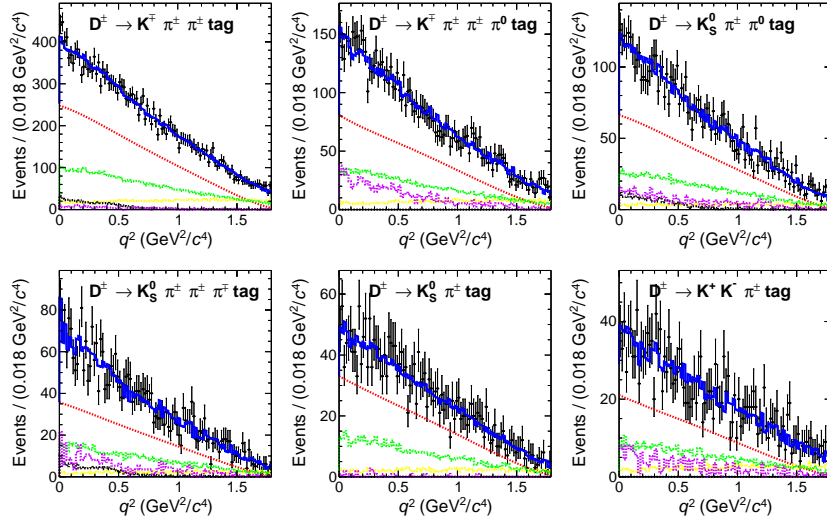


FIG. 6. (Color online) Simultaneous fit to the numbers of DT candidates as a function of q^2 with the two-parameter series expansion parametrization. The points are data and the curves are the fit to data. In each plot, the violet, yellow, green, and black curves refer to Bkg I, Bkg II, Bkg III+Bkg IV, and Bkg V, respectively. The red dashed curve shows the contribution of signal, and the blue solid curve shows the sum of background and signal.

and ISGW2 model to simulate the main possible semi-leptonic and semi-muonic backgrounds. We simultaneously fit the the distributions of observed DT candidates as a function q^2 again. The differences between the form-factor parameters obtained from the two determinations are taken as the systematic uncertainties of the form-factor parameters.

Systematic uncertainties associated with the product $f_+^K(0)|V_{cs}|$ are one half of the systematic uncertainties in the branching fraction measurements, presented in Sec. IV, combined in quadrature with the uncertainties associated with D^+ lifetime (0.67%) [13] and the integration I , which are obtained by varying the form-factor parameters by $\pm 1\sigma$. The systematic uncertainties of $f_+^K(0)|V_{cs}|$ are obtained for the simple pole model, modified pole model, two-parameter series expansion and three-parameter series expansion to be 1.4%, 1.5%, 1.5%, 1.2%, respectively.

The fit results are given in Table IV. As a comparison, Table IV also lists the corresponding form-factor results determined for $D^+ \rightarrow K_S^0 e^+ \nu_e$ from CLEO- c [15]. Our results are consistent with those from CLEO- c within uncertainties except for three-parameter series expansion model due to heavy backgrounds in this analysis. In general, as long as the normalization and at least one shape parameter are allowed to float, all models describe the data well. We choose the two-parameter series fit to determine $f_+^K(0)$ and $|V_{cs}|$.

The BESIII experiment has recently reported the most precise value of $f_+^K(0)|V_{cs}|$ using the two-parameter series expansion for $D^0 \rightarrow K^- e^+ \nu_e$ [21]. It is in agreement with the results reported here.

D. Determination of $f_+^K(0)$ and $|V_{cs}|$

Using the $f_+^K(0)|V_{cs}|$ value from the two-parameter series expansion fit and $|V_{cs}| = 0.97343 \pm 0.00015$ from PDG fits assuming CKM unitarity [13] or $f_+^K(0) = 0.747 \pm 0.019$ from the unquenched LQCD calculation [20] as input, we obtain

$$f_+^K(0) = 0.748 \pm 0.007 \pm 0.012 \quad (25)$$

and

$$|V_{cs}| = 0.975 \pm 0.008 \pm 0.015 \pm 0.025, \quad (26)$$

where the uncertainties are statistical, systematic, and external (in Eq. (26)). For Eq. (25), the external error is negligible (0.0002) compared to our measurement. The measured $f_+^K(0)$ is consistent with the one measured with $D^+ \rightarrow K_S^0 e^+ \nu_e$ at CLEO- c [15]; it is also in good agreement with LQCD predictions, although the currently available LQCD results have relatively large uncertainties. The measured $|V_{cs}|$ is in agreement with that reported by the PDG.

VI. SUMMARY

In this paper we present the first measurement of the absolute branching fraction $\mathcal{B}(D^+ \rightarrow K_L^0 e^+ \nu_e) = (4.481 \pm 0.027(\text{stat.}) \pm 0.103(\text{sys.}))\%$, which is in excellent agreement with $\mathcal{B}(D^+ \rightarrow K_S^0 e^+ \nu_e)$ measured by CLEO- c [15]. The CP asymmetry $A_{CP}^{D^+ \rightarrow K_L^0 e^+ \nu_e} = (-0.59 \pm 0.60(\text{stat.}) \pm 1.48(\text{sys.}))\%$, which agrees with theoretical prediction on CP violation in K^0 system within the

TABLE IV. Comparison of results of $f_+^K(0)|V_{cs}|$ and shape parameters (m_{pole} , α , r_1 and r_2) to previous corresponding results determined by $D^+ \rightarrow K_S^0 e^+ \nu_e$ from CLEO-c [15]. The first uncertainties are statistical, and the second are systematic.

Single pole model			
Decay mode	$f_+^K(0) V_{cs} $	m_{pole} (GeV/ c^2)	
$D^+ \rightarrow K_L^0 e^+ \nu_e$	$0.729 \pm 0.006 \pm 0.010$	$1.953 \pm 0.044 \pm 0.036$	
$D^+ \rightarrow K_S^0 e^+ \nu_e$	$0.720 \pm 0.006 \pm 0.009$	$1.95 \pm 0.03 \pm 0.01$	
Modified pole model			
Decay mode	$f_+^K(0) V_{cs} $	α	
$D^+ \rightarrow K_L^0 e^+ \nu_e$	$0.727 \pm 0.006 \pm 0.011$	$0.239 \pm 0.077 \pm 0.065$	
$D^+ \rightarrow K_S^0 e^+ \nu_e$	$0.715 \pm 0.007 \pm 0.009$	$0.28 \pm 0.06 \pm 0.02$	
Two-parameter series expansion			
Decay mode	$f_+^K(0) V_{cs} $	r_1	
$D^+ \rightarrow K_L^0 e^+ \nu_e$	$0.728 \pm 0.006 \pm 0.011$	$-1.91 \pm 0.33 \pm 0.28$	
$D^+ \rightarrow K_S^0 e^+ \nu_e$	$0.716 \pm 0.007 \pm 0.009$	$-2.10 \pm 0.25 \pm 0.08$	
Three-parameter series expansion			
Decay mode	$f_+^K(0) V_{cs} $	r_1	r_2
$D^+ \rightarrow K_L^0 e^+ \nu_e$	$0.737 \pm 0.006 \pm 0.009$	$-2.23 \pm 0.42 \pm 0.53$	$11.3 \pm 8.5 \pm 8.7$
$D^+ \rightarrow K_S^0 e^+ \nu_e$	$0.707 \pm 0.010 \pm 0.009$	$-1.66 \pm 0.44 \pm 0.10$	$-14 \pm 11 \pm 1$

statistical error, is also determined. By fitting the distributions of the observed DT events as a function of q^2 , $f_+^K(0)|V_{cs}|$ and the corresponding parameters for three different theoretical form-factor models are determined. Taking $f_+^K(0)|V_{cs}|$ from the two-parameter series expansion parametrization, $f_+^K(0)|V_{cs}| = 0.728 \pm 0.006(\text{stat.}) \pm 0.011(\text{sys.})$ and using $|V_{cs}|$ from the SM constraint fit, we find $f_+^K(0) = 0.748 \pm 0.007(\text{stat.}) \pm 0.012(\text{sys.})$. By using an unquenched LQCD prediction for $f_+^K(0)$, $|V_{cs}| = 0.975 \pm 0.008(\text{stat.}) \pm 0.015(\text{sys.}) \pm 0.025(\text{LQCD})$.

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Appendix A: Systematic uncertainty in K_L^0 reconstruction efficiency

To determine the systematic uncertainty in the K_L^0 reconstruction efficiency, we measure the K_L^0 efficiency in data and MC using a partial reconstruction technique. We then determine the efficiency difference between data and MC, $\epsilon_{\text{data}}/\epsilon_{\text{MC}} - 1$, of the K_L^0 reconstruction efficiency, where ϵ_{MC} is the efficiency for MC and ϵ_{data} is the efficiency for data.

Based on 1.3 B J/ψ events collected by BESIII detector in years 2009 and 2012, we use two control samples to measure K_L^0 reconstruction efficiency. One sample is $J/\psi \rightarrow K^*(892)^\pm K^\mp$ with $K^*(892)^\pm \rightarrow K_L^0 \pi^\pm$, and the other is $J/\psi \rightarrow \phi K_L^0 K^\pm \pi^\mp$. We reconstruct all the particles in the event except the K_L^0 whose efficiency we wish to measure. The number of $K^0(\bar{K}^0)$ is denoted by N_1 . Then, by applying K_L^0 selection requirements mentioned in Sec. III B, we obtain the number of $K^0(\bar{K}^0)$ denoted by N_2 . Here, in order to select K_L^0 control samples with low level of backgrounds, we perform the kinematic fit to select K_L^0 candidate with the minimal χ^2 and require $\chi^2 < 100$.

$K^0(\bar{K}^0)$ reconstruction efficiency is calculated by $\epsilon = N_2/N_1$. For data, N_1 , N_2 are determined by fitting the missing mass squared distribution of K_L^0 . Each fit included a signal line shape function which is determined from MC samples smeared with a Gaussian resolution,

and the background shape is determined from MC samples as well. With respect to MC samples, N_1, N_2 are obtained from MC truth directly. The fits are performed in separate momentum bins. In each fit, N_1 (N_2) consists of the number of K_L^0 and K_S^0 . The ratio of K_L^0 to K_S^0 is estimated from MC simulations. Due to the effect of the difference in nuclear interactions of K^0 and \bar{K}^0 mesons, we consider $K^0 \rightarrow K_L^0$ and $\bar{K}^0 \rightarrow K_L^0$ separately. We use the charge of kaon to tag K^0 or \bar{K}^0 in the control sample, which means if we find a K^+ in the process, the corresponding K_L^0 must be derived from \bar{K}^0 .

Figure 7 shows the distributions of K_L^0 reconstruction efficiency differences between data and MC in 19 momentum bins for the processes of $K^0 \rightarrow K_L^0$ and $\bar{K}^0 \rightarrow K_L^0$.

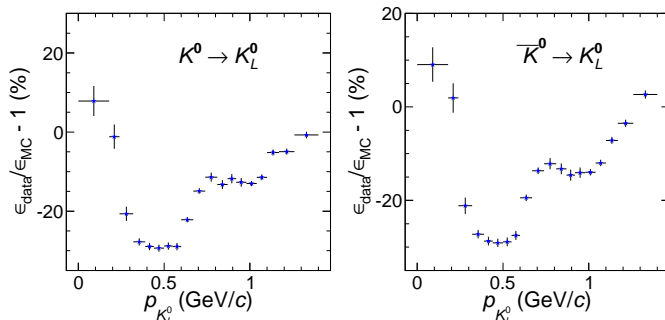


FIG. 7. Distributions of K_L^0 reconstruction efficiency differences between data and MC for the processes of $K^0 \rightarrow K_L^0$ and $\bar{K}^0 \rightarrow K_L^0$.

The probability of an inelastic interaction of a neutral kaon in the detector depends on the strangeness of the kaon at any point along its path, which is due to oscillations in kaon strangeness and different nuclear cross sections for K^0 and \bar{K}^0 . Hence, the total efficiency to observe a final state K_L^0 (K_S^0) differs from that expected for either K^0 or \bar{K}^0 . This effect is related to the coherent regeneration of neutral kaons [22]. However, the detector-simulation program GEANT4 does not take into account this effect. The time-dependent K^0 - \bar{K}^0 oscillations are thereby ignored in GEANT4. Considering the massive detector materials in the outer of the MDC, the TOF counter and the EMC, it results in an obvious discrepancy ($>10\%$) of K_L^0 shower-finding efficiency in the EMC between data and MC. On the other hand, we take the same method to study K_S^0 reconstruction efficiency difference between data and MC for the processes of $K^0 \rightarrow K_S^0$ and $\bar{K}^0 \rightarrow K_S^0$ by 224 M J/ψ control sample, as shown in Fig. 8. We find that the K_S^0 reconstruction efficiency of data is a little higher than that of MC, which gives another hint of the absence of the coherent regeneration of neutral kaons by GEANT4.

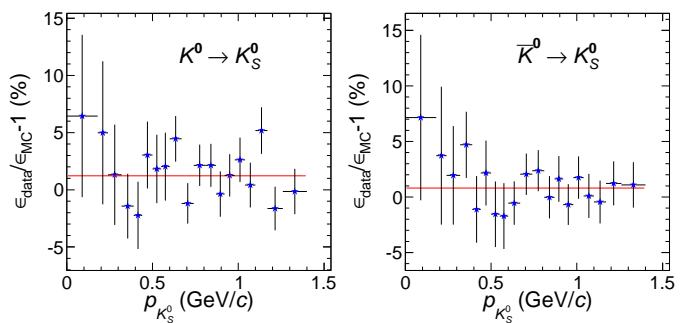


FIG. 8. Distributions of K_S^0 reconstruction efficiency differences between data and MC for the processes of $K^0 \rightarrow K_S^0$ and $\bar{K}^0 \rightarrow K_S^0$. The red line is the fit to the points in the form of zero-order polynomial.

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